Es n°5 - 9/12/13

Impresa → Costo totale
$T : (0, +\infty) \to \mathbb{R}$
$T(x) = x^2 + 10x + 25$

Costo Marginale $M(x) = T'(x)$
$M : (0, +\infty) \to \mathbb{R}$

Costo Medio $N(x) = \frac{T(x)}{x}$
$N : (0, +\infty) \to \mathbb{R}$

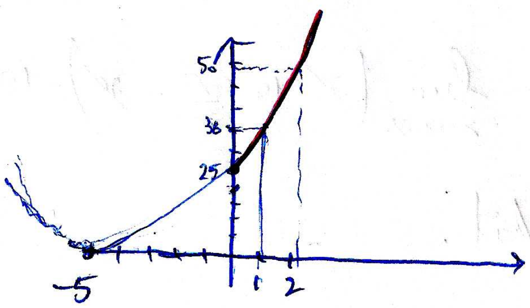
a) Si disegni il grafico delle 3 funzioni

$T(x) = x^2 + 10x + 25$ parabola

$x_v = -\frac{10}{2} = -5$

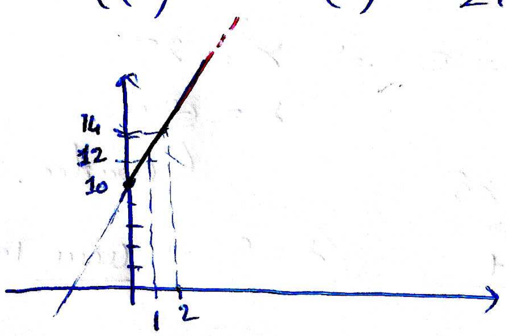
$V(-5; 0)$

$y_v = 25 - 50 + 25 = 0$



$M(x) = T'(x) = 2x + 10$ → retta

x	y
0	10
1	12

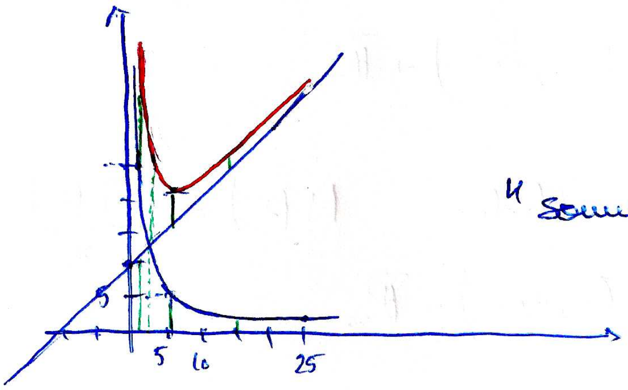


$$N(x) = \frac{x^2+10x+25}{x} = x + 10 + \frac{25}{x}$$

$y_1 = x+10 \rightarrow$ retta $m=1$, $q=10$

$y_2 = \frac{25}{x} \rightarrow$ iperb. equilatera riferita agli asintoti

$V(\sqrt{k}\,;\sqrt{k}) \equiv (5,5)$



"sommo le ordinate"

<u>Opp</u> $y = x+10+\frac{25}{x}$

$\lim_{x\to+\infty} N(x) = +\infty \rightarrow$ Asint. Obl. $y = x+10$

Infatti, Ric. asint. obl.: $m = \lim_{x\to+\infty}\left(1+\frac{10}{x}+\frac{25}{x^2}\right) = 1$

$q = \lim_{x\to+\infty}\left(\cancel{x}+10+\frac{25}{x}-\cancel{x}\right) = 10$

$\lim_{x\to0^+} N(x) = +\infty$

L'asse y è Asint. Vert.

$y' = 1 - \frac{25}{x^2}$

$y' = 0 \Rightarrow \frac{x^2-25}{x^2} = 0 \Rightarrow x^2 = 25$

$x = \pm 5$

Non Acc.

$y' > 0 \Rightarrow x < -5 \cup x > 5$

0 $-$ 5 $+$

$x=5 \rightarrow$ <u>min</u> rel

b) Verif. Costo Marg. e Costo medio sono uguali nel pt di min del C. Medio

C. Medio → ha un min per $x=5$

C Marg = C. Medio

$\Downarrow$

$$2x+\cancel{10} = x+\cancel{10}+\frac{25}{x}$$

$$x-\frac{25}{x}=0$$

$$\frac{x^2-25}{x}=0 \Rightarrow x^2=25 \Rightarrow x=\pm 5$$

ok

Per $x=5$ infatti:

$$M(5)=20$$

$$N(5)=5+10+\frac{25}{5}=20$$

=

c) C Marg > C Medio ?

$$2x+10 > x+10+\frac{25}{x}$$

$$\frac{x^2-25}{x}>0$$

N>0) $x^2-25>0 \Rightarrow x<-5 \cup x>5$

D>0) $x>0$

	-5	0	5	
			−	+
+	−	−	+	N
−	−	+	+	D

Per $x>5$

Es 1 – 9/12/13

$f: \mathbb{R} \to \mathbb{R}$

$f(x) = e^x - 4e^{x/2}$ Im $f = ?$

è una funz. continua $\Rightarrow$ Devo cercare il max e min assoluti

$\Downarrow$

Confronto i valori che la funz. assume agli estremi, con quelli che assume nei pt critici (pt staz. e di non derivabilità)

$$\lim_{x \to -\infty} f(x) = 0.$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} e^x \left[1 - 4e^{-\frac{x}{2}}\right] = \lim_{x \to +\infty} \underbrace{e^x}_{\to +\infty} \Big(1 - \underbrace{\frac{4}{e^{x/2}}}_{\to 0}\Big) = +\infty$$

$\Rightarrow$ E' superiorm. illimitata.

$$y' = e^x - \cancel{4}^{\,2} \cdot \frac{1}{\cancel{2}} e^{x/2} = e^{x/2}\left(e^{x/2} - 2\right)$$

$\boxed{\text{Im} f \ [-4; +\infty)}$

$$y' = 0 \Rightarrow e^{x/2} - 2 = 0$$

$$e^{x/2} = 2 \Rightarrow \frac{x}{2} = \ln 2 \Rightarrow x = 2\ln 2$$

min rel. e ~~ass~~

$$y' > 0 \Rightarrow e^{x/2} - 2 > 0 \Rightarrow x > 2\ln 2$$

		$2\ln 2$	
y	$\searrow$	min rel.	$\nearrow$
y'	$-$		$+$

$$f(2\ln 2) = f(\ln 4) = e^{\ln 4} - 4e^{\frac{2\ln 2}{2}} = 4 - 4 \cdot 2 = -4$$

Es n2 – 9 Dic '13

$$\lim_{x\to 0} \frac{\sqrt{x+1}-e^x}{x+x^2} = \left(\frac{0}{0}\right)$$

$$\overset{(H)}{=} \lim_{x\to 0} \frac{\frac{1}{2\sqrt{x+1}}-e^x}{(1+2x)} = \lim_{x\to 0} \frac{1-2e^x\sqrt{x+1}}{2(1+2x)\sqrt{x+1}} =$$

$$= \frac{1-2}{2} = -\frac{1}{2}.$$

(*) ↳ Opp. senza Teor. Hospital.

$$\overset{(*)}{=} \lim_{x\to 0} \frac{\sqrt{1+x}-e^x}{x+x^2}\cdot\frac{\left(\sqrt{1+x}+e^x\right)}{\sqrt{1+x}+e^x} =$$

$$= \lim_{x\to 0} \frac{1+x-e^{2x}}{x(1+x)\left[\sqrt{1+x}+e^x\right]} =$$

$$= \lim_{x\to 0} \left(\underbrace{\frac{1}{(1+x)\left[\sqrt{1+x}+e^x\right]}}_{(1)\cdot(2)} \cdot \frac{1+x-e^{2x}}{x}\right) =$$

$$= \frac{1}{2}\cdot\lim_{x\to 0} \frac{x-(e^{2x}-1)}{x} = \frac{1}{2}\cdot\lim_{x\to 0}\left[1-\frac{e^{2x}-1}{x}\right] =$$

$$= \frac{1}{2}\cdot\left[1-\lim_{x\to 0} 2\cdot\underbrace{\frac{e^{2x}-1}{2x}}_{\downarrow\, 1}\right] = \frac{1}{2}(1-2) =$$

$$= \frac{1}{2}(-1) = -\frac{1}{2}.$$

Es n3 - Comp 9/12/13

$$f(x) = \begin{cases} x^2 - x + k^2, & x \le 1 \\ \dfrac{2\ln(x^k)}{x-1}, & x > 1 \end{cases}$$

Si determino tutti i valori di $k \in \mathbb{R}$ per i quali $f(x)$ è continua in $x = 1$.

$$f(1) = \cancel{1} - \cancel{1} + k^2 = k^2$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x^2 - x + k^2) = k^2$$

Ponendo $t = x-1$

$$\lim_{x\to 1^+} \frac{2\ln(x^k)}{x-1} = \lim_{x\to 1} \frac{2k\ln x}{x-1} \overset{\downarrow}{=} \lim_{t\to 0} 2k \frac{\ln(t+1)}{t} \rightarrow 1$$

$= 2k$

$\Rightarrow$ Deve essere : $2k = k^2$

$k^2 - 2k = 0$

$k(k-2) = 0 \rightarrow k = 0$, $k = 2$.

Es n° 3B - 19/2/13

$f : [0,2] \to \mathbb{R}$

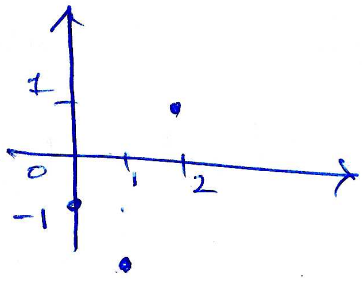
f è cont. in $[0,2]$ e deriv. in $(0,2)$ e

$f(0) = -1$

$f(1) = -2$

$f(2) = 1$

Dim che esiste $c \in (0,2)$ t.c. $f'(c) = 0$



$f(x)$ è cont. in un interv. chiuso e limitato $\Rightarrow$ x Teor. di Weierstrass

$\Downarrow$

Ammette max e min assoluti!

Oss: sicuramente il min assoluto non è negli estremi perchè:

$f(1) < f(0)$

$f(1) < f(2)$

$\Downarrow$

il min assoluto è interno all'interv. $(0,2)$

$\Downarrow$

il min ass. è anche min rel.

$\exists c \in (0,2)$ t.c.

$f(c) \leq f(x) \quad \forall x \in (0,2)$

$\Downarrow$

per $x = c \in (0,2)$ si ha un min relativo $\xRightarrow{\text{x il teor. di Fermat}} f'(c) = 0.$

Es n°5 – Comp. 27/1/09

$\alpha, \beta, \gamma : \mathbb{R} \to \mathbb{R}$ derivabili crescenti e posit. $\forall x \in \mathbb{R}$

$k, \ell > 0$

$$f(x) = \gamma\left(k\alpha(x) - \ell\beta(x)\right)$$
$$g(x) = \ln\left(\beta(x)\cdot\alpha(x)\right)$$
$$h(x) = \cos\left(\alpha(x)\cdot\beta(x)\right)$$

è possibile affermare che le funz. f, g, h sono crescenti?

$f, g, h : \mathbb{R} \to \mathbb{R}.$

$$f'(x) = \overbrace{\gamma'\left(k\alpha(x) - \ell\beta(x)\right)}^{(+)} \cdot \overbrace{\left(k\alpha'(x) - \ell\beta'(x)\right)}^{\overset{(?)}{(+)\cdot(+)\ -\ (+)\cdot(+)}}$$

$\alpha, \beta, \gamma > 0$ perché posit.

$\alpha', \beta', \gamma' > 0$ perché crescenti

Non si può dire!

$$g'(x) = \underbrace{\frac{1}{\underset{(+)\cdot(+)}{\beta(x)\cdot\alpha(x)}}}_{(+)} \cdot \underbrace{\left[\underset{(+)\cdot(+)\ +\ (+)\cdot(+)}{\beta'(x)\cdot\alpha(x) + \beta(x)\alpha'(x)}\right]}_{(+)}$$

$g'(x) > 0 \Rightarrow g(x)$ è crescente!

$$h'(x) = \underbrace{-\sin\left(\underbrace{\alpha(x)\cdot\beta(x)}_{(+)}\right)}_{?} \cdot \left[\underbrace{\alpha'(x)\cdot\beta(x) + \alpha(x)\cdot\beta'(x)}_{(+)}\right]$$

NON si può dire!

Es 5 – Comp 22/4/09

$\alpha, \beta, \gamma : \mathbb{R} \to \mathbb{R}$ deriv. e crescenti

$\Downarrow$

$\beta(x) > 0 \quad \forall x \in \mathbb{R} \qquad \alpha', \beta', \gamma' > 0$

$f, g, h : \mathbb{R} \to \mathbb{R}$

a) $f(x) = \dfrac{\alpha(x) + \beta(x)}{\gamma(x)}$ è crescente ?

$$f'(x) = \frac{\overbrace{(\alpha'(x) + \beta'(x))}^{(+)+(+)} \cdot \overbrace{\gamma(x)}^{?} - \overbrace{\gamma'(x)\,[\alpha(x) + \beta(x)]}^{(+)\cdot(?+(+))}}{\gamma^2(x)}$$

Non si può dire.

b) $g(x) = \alpha(x) + \beta^2(x)$

$$g'(x) = \underbrace{\alpha'(x)}_{(+)} + \underbrace{2\beta(x) \cdot \beta'(x)}_{\underbrace{(+)\cdot(+)}_{+}} > 0 \Rightarrow g(x) \text{ Cresce}$$

c) $h(x) = \cos(\alpha(x) + \gamma(x))$

$$h'(x) = -\underbrace{\operatorname{sen}(\overset{?}{\alpha(x)} + \overset{?}{\gamma(x)})}_{?} \cdot \overbrace{[\alpha'(x) + \gamma'(x)]}^{+}$$

Non si può dire!

Es 2 – Comp. 22/4/09

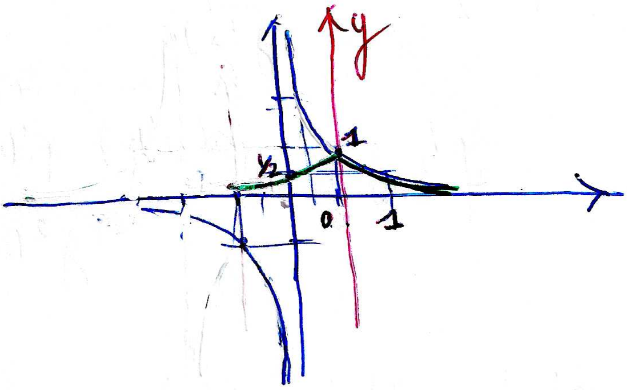
A) $f(x) = -\log\left(\frac{1}{|x|+1}\right)$

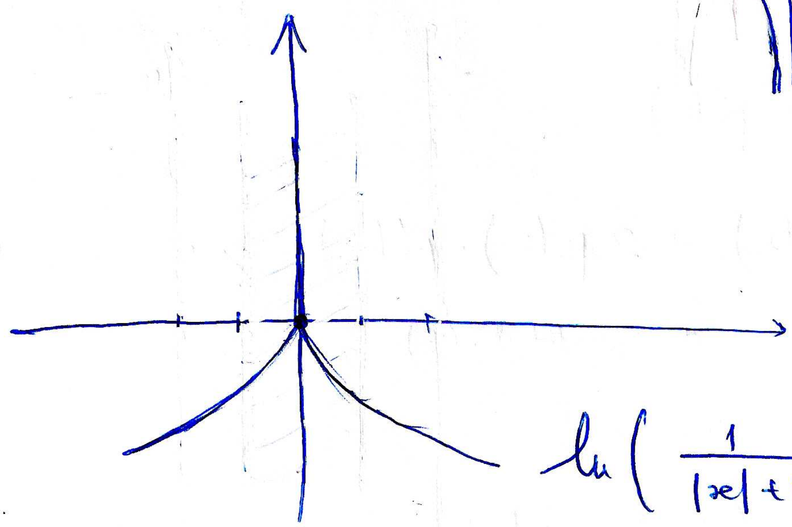
Fare il graf. utilizzando graf. elementari e trasf. geom.

1) $y = \frac{1}{x}$ → iperb. equil.

2) $y = \frac{1}{x+1}$ → trasl. vs

3) $y = \frac{1}{|x|+1}$





$\ln\left(\frac{1}{|x|+1}\right)$

B) $$\lim_{x\to+\infty} \frac{(x+1)(3x+4)}{|x^3-1| - |2x^2-x^3|} = \lim_{x\to+\infty} \frac{3x^2}{+\cancel{x^3}-1-(-2x^2+\cancel{x^3})} =$$

$$= \lim_{x\to+\infty} \frac{3x^2}{2x^2} = \frac{3}{2}$$

$$|x^3-1| = \begin{cases} x^3-1, & x \geq 1 \\ -x^3+1, & x<1 \end{cases}$$

$$|2x^2-x^3| = \begin{cases} 2x^2-x^3, & x \leq 2 \\ -2x^2+x^3 & x>2 \end{cases}$$

Es 4 - Comp 9/6/09

$f(x) = -\sqrt{x+1} + \sqrt{x-1}$ Studiare

Dal graf. dire se ammettono soluz. l'equazione

a) $-\sqrt{x+1} + \sqrt{x-1} = -700$

b) $-\sqrt{x+1} + \sqrt{x-1} + 1,2564 = 0$

Dominio:

$\begin{cases} x+1 \geq 0 \Rightarrow x \geq -1 \\ x-1 \geq 0 \Rightarrow x \geq 1 \end{cases}$ $x \geq 1$

Int. asse y: No perché $x=0 \notin$ Dom.

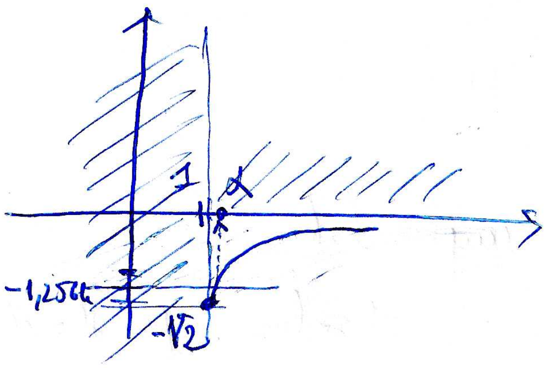
Int. asse x: $\sqrt{x+1} = \sqrt{x-1}$

$\cancel{x}+1 = \cancel{x}-1$ mai

Segno:

$f(x) > 0 \Rightarrow \sqrt{x-1} > \sqrt{x+1}$

$\begin{cases} x-1 \geq 0 \\ x+1 \geq 0 \\ \cancel{x}-1 > \cancel{x}+1 \end{cases}$ $\Big\}$ $x \geq 1$ IMPOSS. $\Rightarrow$ $f(x)$ è sempre negativa.



$f(1) = -\sqrt{2}$

$\lim_{x \to +\infty} f(x) = $ f. ind $-\infty + \infty$

$$\lim_{x\to+\infty} \frac{\left(-\sqrt{x+1}+\sqrt{x-1}\right)\left(\sqrt{x+1}+\sqrt{x-1}\right)}{\sqrt{x+1}+\sqrt{x-1}} =$$

$$= \lim_{x\to+\infty} \frac{x-1-(x+1)}{\sqrt{x+1}+\sqrt{x+1}} = \frac{-2}{+\infty} = 0^-$$

L'asse x è Asint. Orizz.

$$y' = -\frac{1}{2\sqrt{x+1}} + \frac{1}{2\sqrt{x-1}} = \frac{-\sqrt{x-1}+\sqrt{x+1}}{2\sqrt{(x+1)(x-1)}}$$

$$y' = 0 \Rightarrow \sqrt{x-1} = \sqrt{x+1}$$

$$x-1 = x+1$$ imp, No pt Staz.

$$y' > 0 \Rightarrow \sqrt{x+1} > \sqrt{x-1}$$

$$\begin{cases} x+1 \geq 0 \\ x-1 \geq 0 \\ x+1 > x-1 \end{cases} \quad \begin{matrix} x \geq 1 \\ \\ \forall x \in \mathbb{R} \end{matrix} \quad \Rightarrow \quad y' > 0 \ \forall x \in D_f$$

$\Downarrow$

y sempre crescente

$$f'_+(1) = \lim_{x\to 1^+} \frac{-\sqrt{x-1}+\sqrt{x+1}}{2\sqrt{(x+1)(x-1)}} = +\infty$$

Imf : $[-\sqrt{2}\,;0)$

a) $-\sqrt{x+1}+\sqrt{x-1} = -700$ impossibile

b) $-\sqrt{x+1}+\sqrt{x-1} = -1{,}2564 \Rightarrow$ 1 soluz. per $x=\alpha$.

Es 1 - Comp. 9/7/09

$f(x) = \left| 1 - \ln|x| \right|$

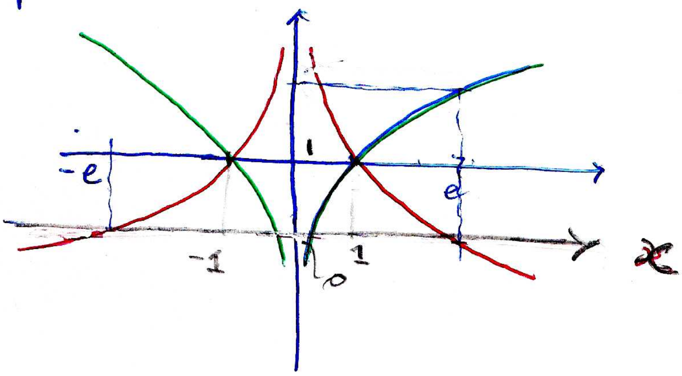
1) $y = \ln x$

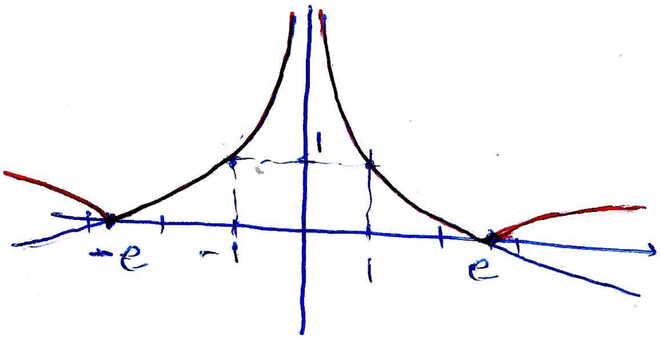
2) $y = \ln|x|$

3) $y = -\ln|x|$

4) $y = -\ln|x| + 1$

5) $y = \left|1 - \ln|x|\right|$





Es 2 - Comp. 9/7/09

$$\lim_{x\to 0} \frac{1-\sqrt{\cos x}}{x \operatorname{sen} x} = \lim_{x\to 0} \frac{(1-\sqrt{\cos x})(1+\sqrt{\cos x})}{x \operatorname{sen} x\,(1+\sqrt{\cos x})} =$$

$$= \lim_{x\to 0} \frac{1-\cos x}{x \operatorname{sen} x\,(1+\sqrt{\cos x})} =$$

$$= \lim_{x\to 0} \underbrace{\frac{1-\cos x}{x\cdot x}}_{\downarrow\ \frac{1}{2}} \cdot \underbrace{\frac{x}{\operatorname{sen} x}}_{\downarrow\ 1} \cdot \underbrace{\frac{1}{1+\sqrt{\cos x}}}_{\downarrow\ \frac{1}{1+1}} = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}.$$

Es n° 156 (Eserciziario)

Dim. $x^5+2x-1=0$ ammette una soluz. in $(0,1)$ usando il Teor. degli zeri

Si dim. che tale soluz. è unica e se ne determini una sua appross. a meno di $1/10$.

* Teor. zeri:

$f(x)$ cont. in $[0,1]$

$$\left.\begin{array}{l} f(0)=-1<0 \\ f(1)=\cancel{1}+2-\cancel{1}=2>0 \end{array}\right\} \exists\, c\in(0,1) \text{ tc } f(c)=0$$

$f'(x)=5x^4+2 > 0 \quad \forall x\in\mathbb{R}$

$\Rightarrow$ in $(0,1)$ la funz. è sempre crescente

$\Rightarrow \exists!\, c\in(0,1) : f(c)=0$

Err $< 1/10$

n° iteraz. necess. met Bisez.

$$k \geq \log_2 \frac{1}{1/10} - 1$$

$$k \geq 3{,}32 - 1$$

$$k \geq 2{,}32 \longrightarrow \text{Almeno 3 iterazioni}$$

$x^5+2x-1=0$ in $(0,1)$

Passo iniz.
$k=0$

$\left.\begin{array}{l} a_0=0 \\ b_0=1 \end{array}\right\} \quad c_1=\frac{0+1}{2}=\frac{1}{2}$

$Err \leq \frac{b-a}{2^{k+1}}=\frac{1}{2}$

$f(0)=-1 \quad (<0)$

$f(1/2)=0{,}03125=\frac{1}{32} \quad (>0)$

$f(1)=2$

I_1

I iteraz.
$k=1$
$\left(Err \leq \frac{1}{4}\right)$

$\left.\begin{array}{l} a_1=0 \\ b_1=1/2 \end{array}\right\} \quad c_2=\frac{1/2}{2}=1/4$

$f(0)<0$

$f(1/4)=(1/4)^5+2\cdot\frac{1}{4}-1=-\frac{511}{1024} \quad (<0)$

$f(1/2) \quad (>0)$

I_2

II iteraz.:
$k=2$
$Err \leq \frac{1}{8}$

$\left.\begin{array}{l} a_2=1/4 \\ b_2=1/2 \end{array}\right\} \quad c_3=\frac{+\frac{1}{4}+\frac{1}{2}}{2}=\frac{3}{8}$

I_3

$f(1/4)<0$

$f(3/8)=(3/8)^5+2\cdot\frac{3}{8}-1=-\frac{7949}{32'768} \quad <0$

$f(1/2)>0$

III iteraz.
$k=3$
$Err \leq \frac{1}{16} \left(< \frac{1}{10}\right)$

$\left.\begin{array}{l} a_3=3/8 \\ b_3=1/2 \end{array}\right\} \quad c_4=\frac{3/8+1/2}{2}=\frac{7}{16} \simeq 0{,}4375$

x^*

$f(7/16)=-0{,}10897\ldots$

$c_4-\frac{1}{16} \leq x^* \leq c_4+\frac{1}{16}$

$0{,}375 \leq x^* \leq 0{,}5 \qquad (x^* \simeq 0{,}486389\ldots)$