COMPITO di ECONOMIA – 7/7/14

(1)

$$f(x) = kx - |x| = \begin{cases} kx - x, & x \geq 0 \\ kx + x, & x < 0 \end{cases}$$

$$f(x) = \begin{cases} (k-1)x, & x \geq 0 \\ (k+1)x, & x < 0 \end{cases}$$

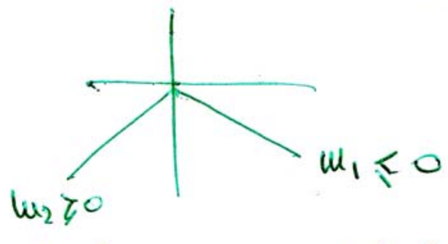
$\forall k$ si tratta di 2 semirette uscenti dall'orig. $y = mx$

$$f'(x) = \begin{cases} k-1, & x \geq 0 \\ k+1, & x < 0 \end{cases}$$

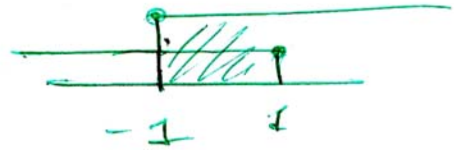
$$\begin{cases} m_1 = k-1 & (x \geq 0) \\ m_2 = k+1 & (x < 0) \end{cases}$$

Si ha un max globale se:

$$\begin{cases} m_2 \geq 0 \\ m_1 \leq 0 \end{cases} \Rightarrow \begin{cases} k+1 \geq 0 \\ k-1 \leq 0 \end{cases} \Rightarrow \begin{cases} k \geq -1 \\ k \leq +1 \end{cases}$$

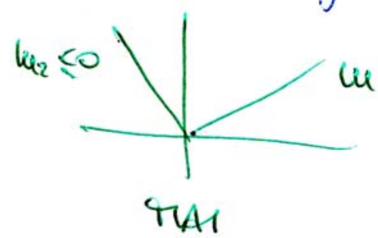


$\longrightarrow$



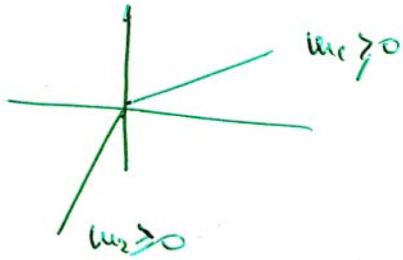
$\boxed{-1 \leq k \leq 1}$

Si ha un min globale se:



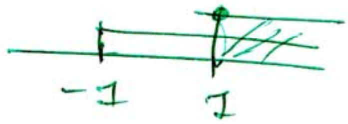
$$\Longrightarrow \begin{cases} k-1 \geq 0 \\ k+1 \leq 0 \end{cases} \Rightarrow \begin{cases} k \geq 1 \\ k \leq -1 \end{cases} \quad \emptyset$$

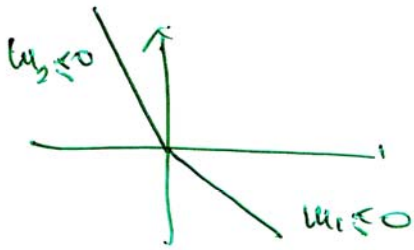
$f(x)$ è crescente se:



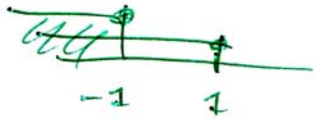
$$\begin{cases} k-1 \geq 0 \\ k+1 \geq 0 \end{cases} \Rightarrow \begin{matrix} k \geq 1 \\ k \geq -1 \end{matrix} \qquad (\boxed{k \geq 1})$$

$f(x)$ è decrescente se:





$$\begin{cases} k-1 \leq 0 \\ k+1 \leq 0 \end{cases} \Rightarrow \begin{cases} k \leq 1 \\ k \leq -1 \end{cases} \qquad (\boxed{k \leq -1})$$



$\boxed{6}$ Studiare la seguente funzione:

$$f(x) = x\left(\ln^2 x - 3\right)$$

Compito 17/6/14

Dominio: $x > 0$ ← Argomento del ln.

Inters. con asse x:

Si pone $y = 0 \Rightarrow x\left(\ln^2 x - 3\right) = 0$

$x = 0$ non Acc., $\notin$ Dom.

$\ln^2 x = 3$

$\ln x = \pm\sqrt{3}$

$x_1 = e^{\sqrt{3}}$ $(\simeq 5{,}65)$

$x_2 = e^{-\sqrt{3}}$ $(\simeq 0{,}177)$

Attenzione:
$\ln^2 x = (\ln x)^2 =$
$\neq$ $= (\ln x)(\ln x)$
$\ln x^2 = 2\ln x$

- Studio del segno di $f(x)$: $f(x) > 0$

$\Rightarrow x\left(\ln^2 x - 3\right) > 0$

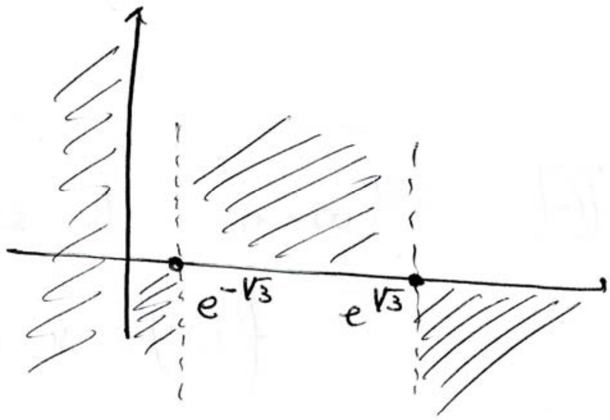
I fattore $> 0 \Rightarrow x > 0$

II fattore $> 0 \Rightarrow \ln^2 x > 3 \xrightarrow{t=\ln x} t^2 > 3$

per i valori esterni

$\ln x < -\sqrt{3} \cup \ln x > \sqrt{3} \Leftarrow t < -\sqrt{3} \cup t > \sqrt{3}$

$0 < x < e^{-\sqrt{3}} \cup x > e^{\sqrt{3}}$



- Studio del comportamento della funz. agli estremi del Dominio ;

$$\lim_{x\to 0^+} \underset{0\cdot+\infty}{x\left(\ln^2 x - 3\right)} = \lim_{x\to 0^+}\left(x\cdot \ln^2 x\right) - \underbrace{\lim_{x\to 0^+} 3x}_{=0} =$$

$$= \lim_{x\to 0^+} x\cdot \ln^2 x =$$

$$= \lim_{x\to 0^+} \frac{\ln^2 x}{\frac{1}{x}} = \left(\frac{\infty}{\infty}\right) \overset{(H)}{=} \lim_{x\to 0^+} \frac{2\ln x\cdot\frac{1}{x}}{-\frac{1}{x^2}} =$$

$$= \lim_{x\to 0^+}\left(-2\ln x\cdot \frac{1}{\cancel{x}}\cdot x^{\cancel{2}}\right) = -2\underbrace{\lim_{x\to 0^+} x\cdot \ln x}_{(=0)} = 0$$

$$\lim_{x \to +\infty} x\left(\ln^2 x - 3\right) = +\infty$$

$$+\infty \cdot +\infty$$

$\llcorner$ Può esserci un asintoto obliquo del tipo $y = mx + q$

Ricerca dell'eventuale asintoto obliquo:

$$m = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \left(\ln^2 x - 3\right) = +\infty$$

$\Rightarrow$ <u>NON</u> c'è un Asint. Obl.

- Ricerca pt stazionari:

$$y' = \ln^2 x - 3 + \cancel{x}\left(2 \ln x \cdot \frac{1}{\cancel{x}}\right) =$$

$$= \ln^2 x - 3 + 2 \ln x$$

$$y' = 0 \Rightarrow \ln^2 x + 2 \ln x - 3 = 0$$

Ponendo $t = \ln x$ si ha:

$$t^2 + 2t - 3 = 0$$

$$\Delta/4 = 1 + 3 = 4$$

$$t_{1,2} = -1 \pm 2 < \begin{matrix} 1 \\ -3 \end{matrix}$$

$$\ln x = 1 \Rightarrow \boxed{x = e}$$

$$\ln x = -3 \Rightarrow \boxed{x = e^{-3}} \; (\simeq 0{,}05)$$

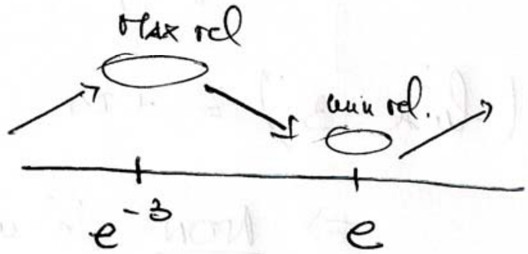
pt. Stazionari.

$$y' > 0 \Rightarrow t^2 + 2t - 3 > 0$$

$$t < -3 \quad \cup \quad t > 1$$

$$\ln x < -3 \quad \cup \quad \ln x > 1$$

$$x < e^{-3} \quad \cup \quad x > e \rightarrow \text{f. cresce}$$



per $x = e^{-3} = \frac{1}{e^3}$ si ha un pt di max rel

$$e \quad f\left(\frac{1}{e^3}\right) = \frac{1}{e^3} \cdot \left(\ln^2 e^{-3} - 3\right) =$$

$$= \frac{1}{e^3} \cdot \left[\left(\ln e^{-3}\right)^2 - 3\right] =$$

$$= \frac{1}{e^3} \left[\left(-3 \underset{(=1)}{\ln e}\right)^2 - 3\right] =$$

$$= \frac{1}{e^3} \cdot \left(9 - 3\right) = \frac{6}{e^3} \quad (\simeq 0{,}3)$$

per $x = e$ si ha un pt di min rel e

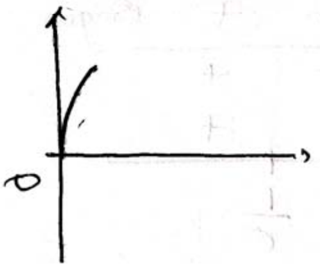
$$f(e) = e\left[(\ln e)^2 - 3\right] = e\,(1 - 3) =$$

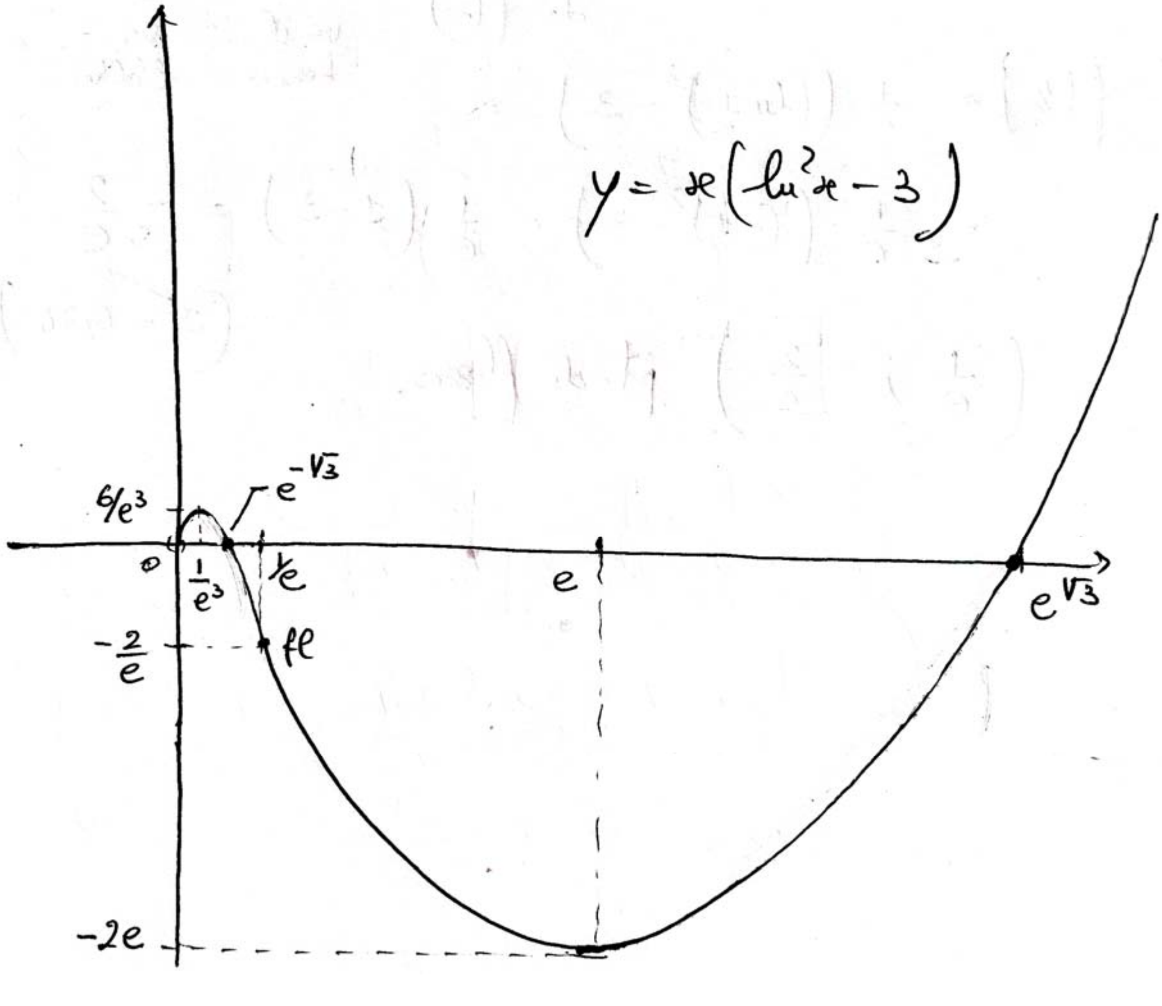
$$= -2e \quad (\simeq -5{,}44)$$

Oss. Vediamo con quale pendenza la curva parte da $x=0$.

$$\lim_{x\to 0^+} y' = \lim_{x\to 0^+} \left(\underbrace{\ln^2 x}_{+\infty} + \underbrace{2\ln x}_{-\infty} - 3 \right) =$$

$$= \lim_{x\to 0^+} \underbrace{\ln x}_{(-\infty)} \cdot \left(\underbrace{\ln x}_{(-\infty)} + 2 - \underbrace{\frac{3}{\ln x}}_{\left(\frac{3}{-\infty}\to 0\right)} \right) = +\infty$$





– Studio della concavità e dei flessi.

$$y'' = 2\ln x \cdot \frac{1}{x} + 2 \cdot \frac{1}{x} = \frac{2}{x}\left(\ln x + 1\right)$$

$$y'' = 0 \Rightarrow \ln x = -1 \Rightarrow x = e^{-1} = \frac{1}{e}$$

$(\simeq 0{,}368)$

pt. di flesso

$y'' > 0$

$x > 0$

$\ln x > -1 \Rightarrow x > 1/e$

y''		$\ominus$		$\oplus$
	0	$-$	$\frac{1}{e}$	$+$
		$+$		$+$

Concavità di $f(x)$: $\cap$ vs il basso; $\cup$ vs l'alto.

$$f(1/e) = \frac{1}{e}\left(\left(\ln \frac{1}{e}\right)^2 - 3\right) =$$

$$= \frac{1}{e} \cdot \left((-1)^2 - 3\right) = \frac{1}{e}\left(1 - 3\right) = -\frac{2}{e}$$

$(\simeq -0{,}74)$

$\left(\frac{1}{e} ; -\frac{2}{e}\right)$ pt di flesso.

Compito 7/7/14

2) $$f(x) = \begin{cases} a \log x , & x > 3 \\ \dfrac{b}{x} , & 0 < x \leq 3 \end{cases}$$

Det. a e b in modo che risulti:

i) continua in $x = 3$

$$\lim_{x \to 3^-} f(x) = f(3) = \frac{b}{3}$$

$$\lim_{x \to 3^+} f(x) = a \log 3$$

$\Rightarrow$ Deve essere: $a \cdot \log 3 = \dfrac{b}{3}$

$$\boxed{b = 3 \log 3 \cdot a}$$

per essere derivabile deve essere continua!

ii) Derivabile in $x_0 = 3$

$$f'_+(3) = \lim_{h \to 0^+} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0^+} \frac{a \log(3+h) - b/3}{h} =$$

$$= \lim_{h \to 0^+} \frac{a \log(3+h) - \log 3 \cdot a}{h} \overset{(H)}{=} \lim_{h \to 0^+} a \cdot \frac{1}{3+h} = \frac{a}{3}$$

$\boxed{b = -3a}$ $\Downarrow$ IMPOSS.

$$f'_-(3) = \lim_{h \to 0^-} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0^-} \frac{\frac{b}{3+h} - \frac{b}{3}}{h} =$$

$$= \lim_{h \to 0^-} \frac{\cancel{3b} - \cancel{3b} - bh}{3(3+h) \cdot h} = \lim_{h \to 0^-} -\frac{b\cancel{h}}{3(3+h)\cancel{h}} = -\frac{b}{9}$$

Compito 7/7/14

3) $$\lim_{x\to 0^+} \frac{x^2-1-(\sqrt{x}-1)^3}{(\sqrt{x}+2)^2-4} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0^+} \frac{x^2-\cancel{1}-(\sqrt{x^3}-\cancel{1}-3x+3\sqrt{x})}{x+\cancel{4}+4\sqrt{x}-\cancel{4}} = -\frac{3}{4}$$

Considero i termini di grado min

$$\lim_{x\to +\infty} \frac{e^{x^2}+\log x}{\sqrt{e^{x^4}+1}} = \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to +\infty} \frac{e^{x^2}}{\sqrt{e^{x^4}+1}} + \lim_{x\to +\infty} \frac{\log x}{\sqrt{e^{x^4}+1}} =$$

$\hookrightarrow (e^{x^4})^{1/2} = e^{\frac{1}{2}x^4}$

$$= \underbrace{\lim_{x\to +\infty} \frac{e^{x^2}}{e^{\frac{x^4}{2}}}} + \underbrace{\lim \frac{\log x}{e^{\frac{1}{2}x^4}}} = 0.$$

Cresce più rapidam. il Denominatore $\Rightarrow 0$

$$\lim_{x\to +\infty} e^{x^2-\frac{x^4}{2}} = (e^{-\infty}) = 0$$

Compito 7/7/14

4) Taylor $\longrightarrow$ nel pt $x_0 = 0$ fino al 2 grado.

$$f(x) = (8-2x)^{1/3}$$

$R_2(x)$

$$f(x) = f(x_0) + f'(x_0)\cdot x + f''(x_0)\cdot \frac{x^2}{2!} + R_2(x)$$

$$f(0) = \sqrt[3]{8} = 2$$

$$f'(x) = \frac{1}{3}(8-2x)^{-2/3}\cdot(-2)$$

$$f'(0) = -\frac{2}{3}\cdot\frac{1}{\sqrt[3]{\underbrace{8^2}_{(2^3)^2}}} = -\frac{2}{3}\cdot\frac{1}{\sqrt[3]{2^6}} = -\frac{2}{3}\cdot\frac{1}{2^2} = -\frac{1}{6}$$

$$f''(x) = -\frac{2}{3}\cdot\left(-\frac{2}{3}\right)(8-2x)^{-5/3}\cdot(-2) =$$

$$= -\frac{8}{9}(8-2x)^{-5/3} = -\frac{8}{9\sqrt[3]{(8-2x)^5}}$$

$$f''(0) = -\frac{8}{9\cdot\sqrt[3]{2^{15}}} = -\frac{8}{9\cdot 2^5} = -\frac{\cancel{8}}{9\cdot \cancel{32}_{4}} = -\frac{1}{36}.$$

$$f(x) \approx 2 - \frac{1}{6}x - \frac{1}{72}x^2 + R_2(x)$$

$$\lim_{x\to 0}\frac{R_2(x)}{x^2} = 0.$$

$10^{1/3}$ → si trovi un'approssimazione usando lo sviluppo di Taylor di $f(x) = (8-2x)^{1/3}$

$x = -1$

$$10^{1/3} \simeq 2 + \frac{1}{6} - \frac{1}{72} = \frac{155}{72}$$

Es. 26/6/09

$$f(x) = \begin{cases} \cos\left(2 + x^2 + \ln(x+1)\right), & x \geq 0 \\ \cos\left(\sqrt{x^2+4} + x\right), & x < 0 \end{cases}$$

Dopo aver controllato che è continua in $x = 0$ dire se il graf. ammette una retta tg nel pt $(0; f(0))$ e scrivere l'eq. di tale retta.

$$f(0) = \cos(2)$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left[\cos\left(2 + x^2 + \ln(x+1)\right)\right] = \cos 2$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \cos\left(\sqrt{x^2+4} + x\right) = \cos 2.$$

$\Rightarrow$ È continua per $x = 0$

Il graf. ammette tg se è derivabile per $x = 0$

Essendo continua posso applicare il crit. di derivabilità

$\Rightarrow$ È derivabile se: $\lim\limits_{x \to 0^-} f'(x) = \lim\limits_{x \to 0^+} f'(x) = l$

$$f'(x) = \begin{cases} -\operatorname{sen}\left[2 + x^2 + \ln(x+1)\right] \cdot \left(2x + \dfrac{1}{x+1}\right), & x \geq 0 \\ -\operatorname{sen}\left(\sqrt{x^2+4} + x\right) \cdot \left(\dfrac{\cancel{2}x}{\cancel{2}\sqrt{x^2+4}} + 1\right), & x < 0 \end{cases}$$

$$\left.\begin{array}{l} \lim\limits_{x \to 0^+} f'(x) = -\operatorname{sen} 2 \\ \lim\limits_{x \to 0^-} f'(x) = -\operatorname{sen} 2 \end{array}\right\} \underline{\text{ok}} \Rightarrow f'(0) = -\operatorname{sen} 2.$$

Il pt $(0; f(0)) = (0; \cos 2)$

Fascio proprio di centro $P(x_0, y_0)$: $y - y_0 = m(x - x_0)$

$$y - \cos 2 = mx$$

$$m_t = f'(0) = -\operatorname{sen} 2 \Rightarrow t) \; y - \cos 2 = -\operatorname{sen} 2 \cdot x$$

$$y = -\operatorname{sen} 2 \, x + \cos 2.$$

COMPITO 9/7/2009

① $f(x) = |1 - \log|x||$

Utilizzando grafici elementari e semplici trasf. geometriche, si tracci il graf. di $f(x)$

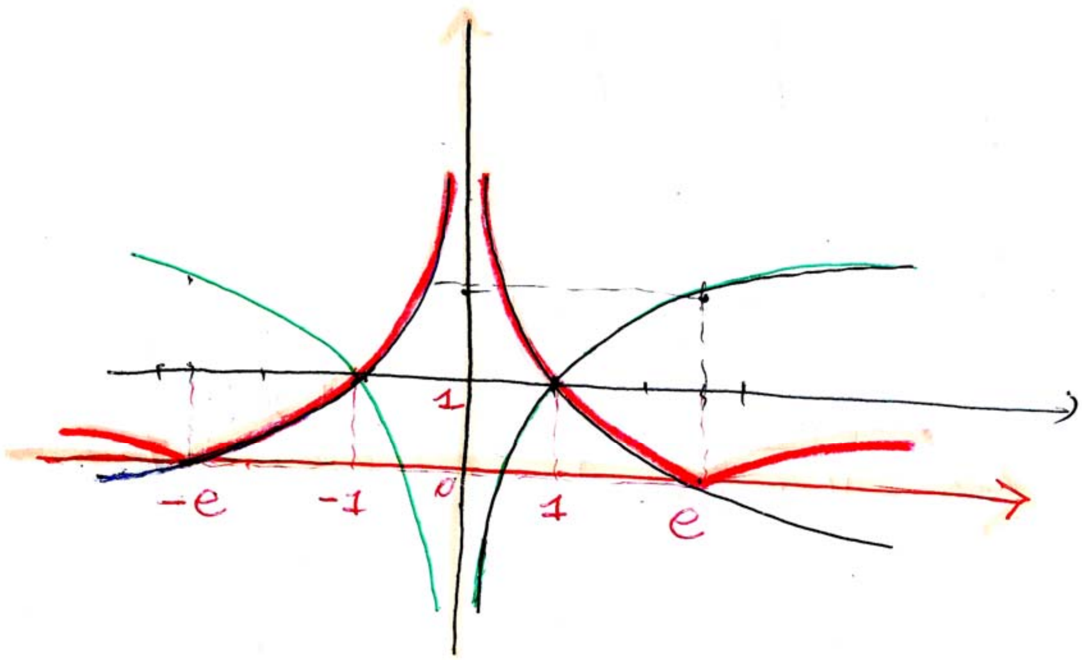
1) $y = \ln x$

2) $y = \ln|x|$

3) $y = -\ln|x|$

4) $y = -\ln|x| + 1$

→ 5) $y = |1 - \ln|x||$



Compito 11/9/14

6) $F(x) = x^2 - 3x$

si dim. tramite def. che $F'(x_0) = 2x_0 - 3$

$$\lim_{h\to 0} \frac{F(x_0+h) - F(x_0)}{h} = F'(x_0)$$

$$\Rightarrow F'(x_0) = \lim_{h\to 0} \frac{(x_0+h)^2 - 3(x_0+h) - (x_0^2 - 3x_0)}{h} =$$

$$= \lim_{h\to 0} \frac{\cancel{x_0^2} + h^2 + 2x_0 h - \cancel{3x_0} - 3h - \cancel{x_0^2} + \cancel{3x_0}}{h} =$$

$$= \lim_{h\to 0} \frac{\cancel{h}(h + 2x_0 - 3)}{\cancel{h}} = 2x_0 - 3$$

c.v.d.

$\boxed{b}$ $D[g(f(x))] = g'(f(x)) \cdot f'(x)$

$g(x) = (f(x))^2 \longrightarrow 2 f(x) \cdot f'(x)$

$h(x) = \cos(2 f(x)) \longrightarrow - \operatorname{sen}(2f(x)) \cdot 2 \cdot f'(x)$

$k(x) = \ln(\sin(2x^2)) \longrightarrow \frac{1}{\operatorname{sen}(2x^2)} \cdot \cos(2x^2) \cdot 4x$